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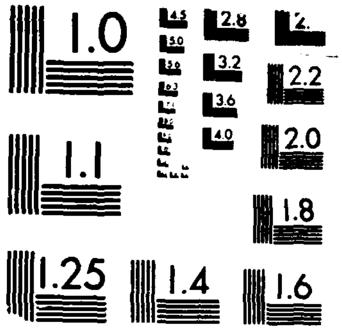

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Contract Report

An Assessment of Modelling Techniques for the Finite Element Analysis of Reinforced Concrete Plate and Shell Structures

ABSTRACT Currently employed modelling techniques used in the analysis of reinforced concrete plate and shell structures are assessed. Methodologies exhibiting short-comings are identified. Particular attention is focused on strain softening, solution algorithms, reinforcement, and invariances. Recommendations concerning future work are made.

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Introduction

Certainly the most useful background reference for finite element modelling of reinforced concrete is the ASCE Task Committee Report [6]. This publication represents the state-of-the-art through 1982.

A recent review article on the finite element analysis of reinforced concrete shell structures describes the current state-of-the-art (Ramm [19]). A main issue seems to be the competition between classical elastic buckling effects and strength, or material modelling, effects. Most reinforced concrete shells in service are very thin compared with structures of interest to NCEL. Thus some issues of interest to the reinforced-concrete shell community are not germane. The relatively thick slab-like magazine structures, with important three-dimensional built-up sections (e.g., around columns), of interest to NCEL make elastic buckling theory largely irrelevant. Issues of material modelling thus become of prime interest as do large-deformation factors due to the intensity of design loadings and concern with ultimate strength. Most currently employed procedures use typical assumptions in the development of shell elements and there seems to be a preference for "degenerated," or continuum-based, shell elements. Plane stress constitutive algorithms are emphasized (see, e.g., Ramm [19, 20]) and variants of the Kupfer bi-axial failure envelope [16] are typically used. In compression and tension, failure mechanisms of crushing and cracking, respectively, are assumed. Both involve strain-softening which is perhaps the most important, delicate, and controversial aspect of concrete modelling. Softening due to cracking is often referred to as tension stiffening. Some form of

smearing is usually employed to model cracking. Typically, elastic moduli are reduced, or set to zero, to manifest the presence of a one or more cracks at a point. Cracks may be assumed to have fixed orientation, or rotate with principal stresses [1, 13]. This is also a controversial aspect of concrete modelling raising issues of invariance and micro-mechanical physical mechanisms [2-4]. To account for rough crack surfaces and doweling some form of shear stiffness retention factor is frequently employed. Tension stiffening is often assumed to subsume phenomena associated with concrete-reinforcement bond slip. This is obviously rather crude at best.

Recent work of Cervera et al. [7,8] is rather advanced in many respects and appears particularly germane to the interests of NCEL. Cervera et al. are concerned with three-dimensional states of stress and especially transverse shear states in the vicinity of supports. They eschew shell elements in favor of three-dimensional finite element models, in particular, 8-20 node isoparametric bricks. They argue against reduced integration procedures and reassess the range of applicability of brick elements in bending situations. This imposes a length-thickness ratio limit in modelling which may be extreme depending on the order of element employed. They model reinforcement by way of perfectly bonded membranes having unidirectional properties "equivalent" to the distribution of steel reinforcing bars. The membrane and concrete brick elements are assembled together in the usual fashion. The assumption of perfect bond is a shortcoming, nevertheless it is typical of the state-of-the-art in large-scale structural modelling. The failure envelope in compression is given by

$$f\ (J_1,\ J_2) = (\alpha J_1 + 3\ \beta J_2)^{1/2} = \sigma_0$$

where J_1 and J_2 are the first and second stress invariants, respectively, and α and β are parameters. These can be used to fit Kupfer's bi-axial data. An associative flow rule is assumed. Hardening is assumed to decrease in this model eventually becoming a perfectly-plastic model. A strain-defined crushing condition, when satisfied, indicates release of all stresses and stiffness. This, of course, is a form of strain softening. Two fixed cracks are allowed to form at each sampling point. After the first crack is formed, the second crack may only form in the plane perpendicular to the first and the formation of the second crack depends only on the two-dimensional state of stress perpendicular to the plane of the first. A cracking failure surface in stress space represents a three-dimensional generalization of the Kupfer-type bi-axial surface in tension. Strain-softening in tension is assumed to obey relations emanating from fracture mechanics which engender mesh-dependent softening moduli which in turn desensitize results to the degree of mesh refinement. These ideas are described in more detail later. It is mentioned that tension stiffening due to reinforcing may be "heuristically included by assuming a higher fracture energy (release rate) for reinforced concrete than for plain concrete." A variable shear retention factor is employed. Cracking is assumed to reduce the compressive hardening modulus in transverse directions. Static as well as dynamic studies have been performed by Cervera et al. [7, 8].

Strain Softening

Cracking and crushing of concrete are frequently modelled with strain-softening mechanisms. From a phenomenological point of view, there is no doubt that softening occurs. However, it is often and convincingly argued that softening is not a constitutive phenomenon, but rather a structural phenomenon indicative of progressive damage. Consider a concrete tensile specimen. In a displacement-controlled test, it is observed experimentally that a peak tensile force is attained and subsequently the force drops off toward zero. This softening is accompanied by a localized band of microcracking and crack coalescence. In the undamaged region of the specimen, elastic unloading takes place. If the softening branch of the force-displacement diagram is interpreted as a material property and used to identify a softening modulus in a constitutive equation, pathological results can ensue. To see this in as simple a setting as possible, assume the softening branch is linear and the one-dimensional stress-strain response is as indicated in Figure 1. The notation is as follows:

σ	=	Tensile stress
σ_f	=	Value of σ at which fracturing commences
ε	=	Tensile strain
ε_f	=	Value of ε at which fracturing commences
ε_0	=	Value of ε at which specimen ceases to carry any load
E	=	Young's modulus
E_s	=	Softening modulus

It is perhaps apparent that the strain computed by dividing the specimen displacement by the specimen length in no way represents the strain of any material point in the specimen. The major portion of the specimen is unstrained due to elastic unloading whereas a small portion has fractured, which may be interpreted as infinite strain. Thus employing an average strain in the development of a constitutive equation clearly is problematic. Nevertheless, this process has been frequently employed in practice. Consider a one-dimensional finite element model of the experiment. Assume a discretization of piecewise linear elements of length h. The constitutive behavior of each element is governed by Figure 1. No matter how many elements are employed, the stable solution to monotonically increasing applied displacement is for one element to follow the stress-strain diagram of Figure 1 and all others to elastically unload as soon as ε exceeds ε_f. This can be seen by imagining that the values of σ_f for each element are randomly perturbed. In this case one element must soften before the others and by virtue of equilibrium the others must experience a decrease in stress prior to ε exceeding their ε_f's. Clearly the solution to the unperturbed problem is non-unique. How well does the solution represent the experiment which was used to derive the constitutive behavior? Let us compute the energy dissipated in the calculation. This is given by the area under the stress-strain diagram multiplied by the volume of the element. Let

A = Cross-sectional area of undamaged specimen.

Then the energy dissipated is

$$\frac{h\,A\,\sigma_f^2}{2}\left(\frac{1}{E}+\frac{1}{E_s}\right) \tag{1}$$

Note that this expression is $O(h)$. Consequently, the results exhibit a mesh sensitivity which is totally spurious. The only way the results of the experiment can be reproduced is if h is taken equal to the specimen length.

This obvious pathology needs to be circumvented otherwise numerical calculations become meaningless. A number of authors have proposed viewing the softening modulus, E_s, as a function of the mesh so that numerical calculations become desensitized. Representative of these works are studies of Willam and his colleagues (see [22 - 24]). The softening modulus is determined so that the energy dissipated, (1), remains constant, viz.,

$$\frac{h\,A\,\sigma_f^2}{2}\left(\frac{1}{E}+\frac{1}{E_s(h)}\right)=A\cdot G_f=\text{constant} \qquad (2)$$

where

G_f = Fracture energy release rate.

From (2),

$$E_s(h)=\frac{E}{\frac{\ell}{h}-1} \qquad (3)$$

where $\ell = 2\,G_f\,E/\sigma_f^2$ denotes a characteristic scale associated with the damaged region.

Remarks

1. As $h \to 0$, $E_s(h) \to 0$.

2. Willam et al. [22] assert that (3) imposes a maximum element length, namely $h \leq \ell$. Although there seems to be no mathematical problem with $h > \ell$ (see Figure 2), typical "strain driven" constitutive algorithms would not be able to handle this situation.

3. Frequently, tensile cracking is modelled with strain-softening elastic-plastic constitutive equations. It is interesting to calculate the plastic modulus, H, corresponding to the softening modulus given by (3). By definition

$$E_s = \frac{-EH}{E + H} \tag{4}$$

and so

$$H(h) = \frac{-h}{\ell} E \tag{5}$$

The mesh-dependence of H is apparent. It appears that this simple way of addressing the mesh sensitivity problem was first proposed by Pietruszczak and Mroz [18] who derived (5) from an entirely different point of view. See also Zimmermann [25] and Willam et al. [23]. Similar ideas have been introduced by Bazant (see, e.g., Bazant and Cedolin [5]), Hillerborg et al. [14], and Nilsson and Oldenburg [17].

4. The generalization of (5) to multidimensional applications is done in ad hoc manner. Various authors have employed (5) with h being taken as simply the square-root of the area of the element in two dimensions and, likewise, the cube-root of the volume in three dimensions (see, e.g., Zimmermann [25]). For this strategy to be reasonable elements need be as close to squares or cubes as possible. Elements with large aspect ratios clearly necessitate h being defined

adaptively as a characteristic element dimension perpendicular to the direction of maximum principal stress. When higher-order, multiple-quadrature-point elements are employed a subelement area or volume may be assigned to each quadrature point for calculating local values of h (see, e.g., Cervera et al. [7, 8]).

5. Modelling fracture damage within the formal structure of classical plasticity theories does not seem consistent with the observation that unloading paths do not in general coincide with those governed by the elastic moduli. Introduction of elastic damage mechanisms causes unloading at reduced values of the elastic modulus. A pure elastic damage model unloads towards the origin (see Fig. 3). The presence of softening creates the same potential for mesh sensitivity within the context of elastic damage mechanisms as described previously. This should be obvious because the illustrated pathology has nothing whatsoever to do with unloading. Recently Resende [21] has proposed an isotropic damage theory for concrete. Despite the presence of softening, no attempt is made in this formulation to desensitize the mesh dependency. Numerical results presented in [21] clearly exhibit this phenomenon. It is of course possible to desensitize an elastic damage model in the same way as described previously. For example, consider the following one-dimensional model:

$$\sigma = (1 - d)\, E\, \varepsilon \tag{6}$$

where $d \in [0,1]$ is the damage parameter. The accumulation of damage is governed by the damage evolution law:

$$\dot{d} = \frac{\hbar}{\tau} E \varepsilon \dot{\varepsilon} \tag{7}$$

Where $\tau = \sqrt{E\varepsilon^2}$ and $\hbar$ is the damage evolution function. (A more detailed description of theories such as this may be found in [15].) Taking the time derivative of (6) and employing (7) yields

$$\dot{\sigma} = \left[(1 - d) E - \frac{\hbar}{\tau} (E\varepsilon)^2 \right] \dot{\varepsilon} \tag{8}$$

The expression in brackets is the tangent modulus during damage loading. At this point $\hbar$ is not specified. It may be determined so that the softening branch of the stress-strain diagram is reproduced as before, viz.,

$$(1 - d) E - \frac{\hbar}{\tau} (E\varepsilon)^2 = - E_S (h) = \frac{-E}{\frac{\ell}{h} - 1} \tag{9}$$

Thus

$$\hbar (h) = \frac{1}{\tau} \left(1 - d + \frac{1}{\frac{\ell}{h} - 1} \right) \tag{10}$$

When a combination of damage mechanisms is employed in a theory, such as in an elastoplastic damage theory [15], it is not so clear how to perform the necessary modifications to desensitize mesh dependency. The ad hoc procedure of introducing the mesh parameter h into constitutive equations thus does not seem satisfactory in more complex situations.

6. Softening branches of stress-strain diagrams are frequently modelled with more complex functions (e.g., exponentials, see Cervera et al. [7, 8], Nilsson and Oldenburg [17]). This poses no essential problems.

Solution Algorithms

Due to turning-point phenomena in solutions of quasi-static reinforced-concrete problems, it is necessary to employ "arc-length" strategies. (These are referred to as "continuation methods" in the mathematics literature.) Crisfield has been a leader in the development and application of these methods to reinforced-concrete structures. Representative papers are [9-12]. Any computer program for the analysis of reinforced-concrete structures should be equipped with algorithms of this type. Unfortunately, even the best procedures encounter difficulties when faced with severe bifurcation and unloading phenomena primarily brought about by softening mechanisms present in reinforced-concrete constitutive models.

Most applications of current interest to NCEL involve dynamic phenomena (e.g., magazines subjected to blast wave loading). Thus state-of-the-art implicit and explicit techniques are required. In addition, due to the complexity of reinforced-concrete behavior, automatic control of time-step size and nonlinear solution strategy within each step, etc., should be included in a computer program. It is felt that a much greater degree of algorithmic automation than present in existing techniques (see, e.g., Cervera et al. [7, 8]) will be desired and, in fact, required for the reliable and efficient solution of problems anticipated.

Reinforcement

Steel reinforcement is modelled as an elastoplastic material in traditional fashion. The most vexing issue in reinforcement modelling is the bond law governing the steel-concrete interface. Studies have been performed in which discrete, nodally assembled reinforcing bars have been modelled and bond-slip laws assumed. These studies have typically been of simple experimental configurations in which, at most, only a few bars are present. Chapter 3 of [6] contains a good summary of work through 1982. Complex reinforcing patterns in engineering structures have been modelled by either distributed and/or embedded equivalent reinforcing elements. Perfect bond is typically assumed. This important aspect of the overall failure strength thus is neglected by state-of-the-art approaches and thus must be considered a serious deficiency of current capabilities in modelling large-deformation failure. There presently does not seem any obvious and rational way of accounting for mechanisms of this type while retaining the computational simplicity of the distributed/imbedded approach.

Invariance

The lack of invariance of the incremental orthotropic tangent moduli approach precludes its use in a complex, dynamic loading environment. Argument for its use despite its defect are simply not convincing. It would seem that the framework provided by anisotropic damage mechanics would be appropriate for formulating invariant constitutive equations capable of representing cracking. The work of Resende [21] is a step in this direction, although it is limited to isotropic mechanisms.

Recommendations

1. The cap model, in its present form [15], does not adequately represent the tension failure surface, tension stiffening, anisotropic elastic damage due to cracking, and crushing. Each of these features can and should be improved. It should be recalled that the cap model has typically been employed in highly-confined, three-dimensional situations and thus the tensile regime has never been an issue. This is not the situation for the plate , slab-like structures of NCEL and these features need to be addressed and improved.

2. Mesh-dependent softening moduli designed to desensitize model response to degree of mesh refinement is an expedient methodology, but one lacking a satisfactory fundamental basis. It is also not clear how to use ideas of this kind when more complex models are utilized (e.g. elastoplastic damage). An investigation into a more fundamental method of developing well-posed models is called for. In the meantime, mesh-dependent softening moduli may be employed, but it should not be lost sight of that it is an ad hoc technology.

3. Damage mechanics should be used as a framework for constitutive theory development so that invariant models may be designed to replace in-place noninvariant models based on incremental orthotropic elastic moduli.

4. The incorporation of bond-slip in distributed/imbedded reinforcement models requires a breakthrough that does not seem imminent. Current developments should employ and be content with the perfect bond hypothesis until new ideas are forthcoming.

5. It is probably worthwhile to pursue a plane-stress, plate-shell model, as well as a more detailed three-dimensional model. The former model will be more useful for large-scale structural modelling , and the latter for qualifying the former and for detailed modelling (e.g., in the vicinity of supports). A new plane-stress reinforced concrete model should be developed. The existing cap should be used as a basis for further development for the three-dimensional constitutive model.

6. Automated arc-length and time-step selection strategies will need to be developed in order to efficiently obtain solutions to reinforced-concrete structural problems. Implicit, unconditionally stable constitutive algorithms should be developed to circumvent sub-incremental stress point stability limitations.

Conclusion

The modelling of reinforced-concrete structures under severe dynamic loading represents a formidable challenge. Many of the key aspects of reinforced concrete are poorly understood, as evidenced by the numerous shortcomings noted in contemporary models. Clearly, one needs to adopt a pragmatic attitude and develop capabilities as best one can in the present and immediate future. Such capabilities will represent a valuable tool for analysis and design of critical structures. At the same time one must not be deceived by the existence of impressive numerical capabilities built upon a foundation lacking in fundamental physical and mathematical understanding. For the foreseeable future, reinforced-concrete modelling capabilities will need to be exercised with extreme care, caution and insight to prevent naive and erroneous engineering judgements from being made. Pragmatic and fundamental research will need to proceed side by side for significant progress to be made.

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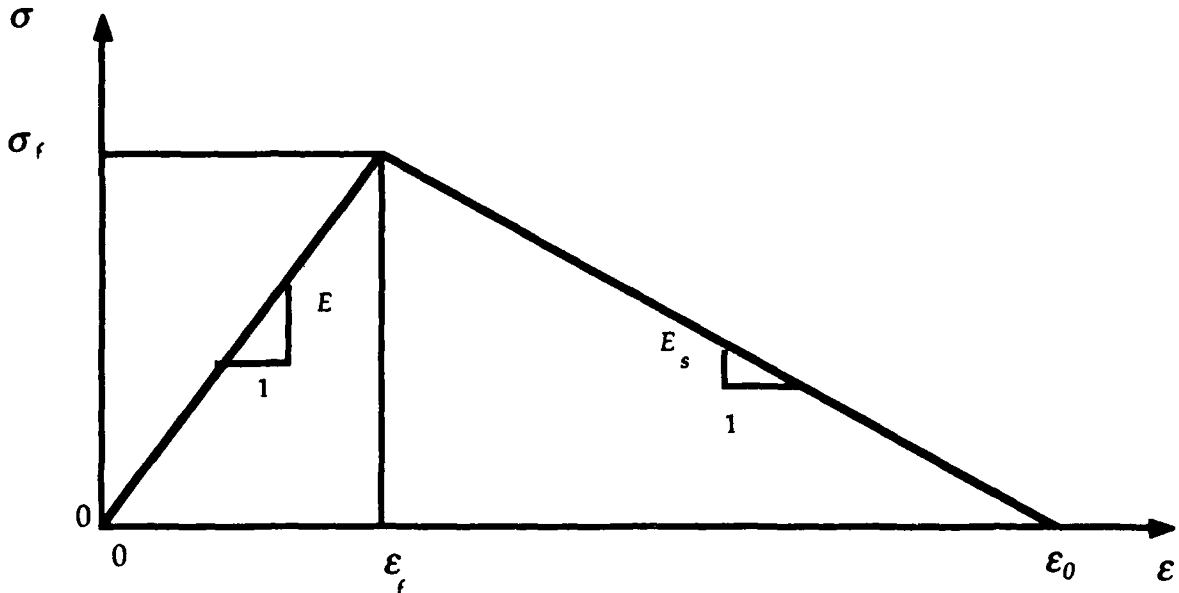

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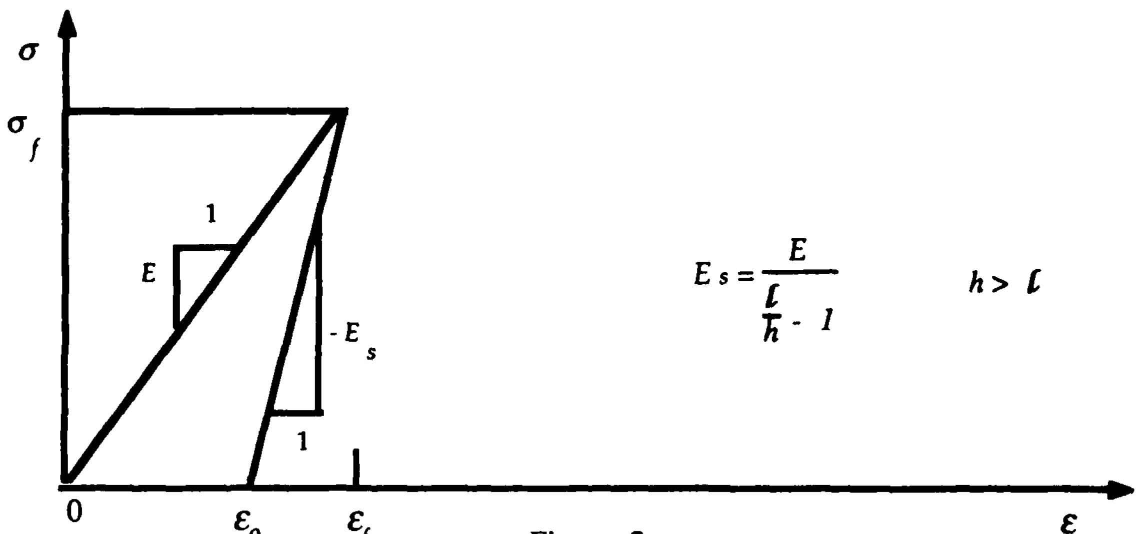

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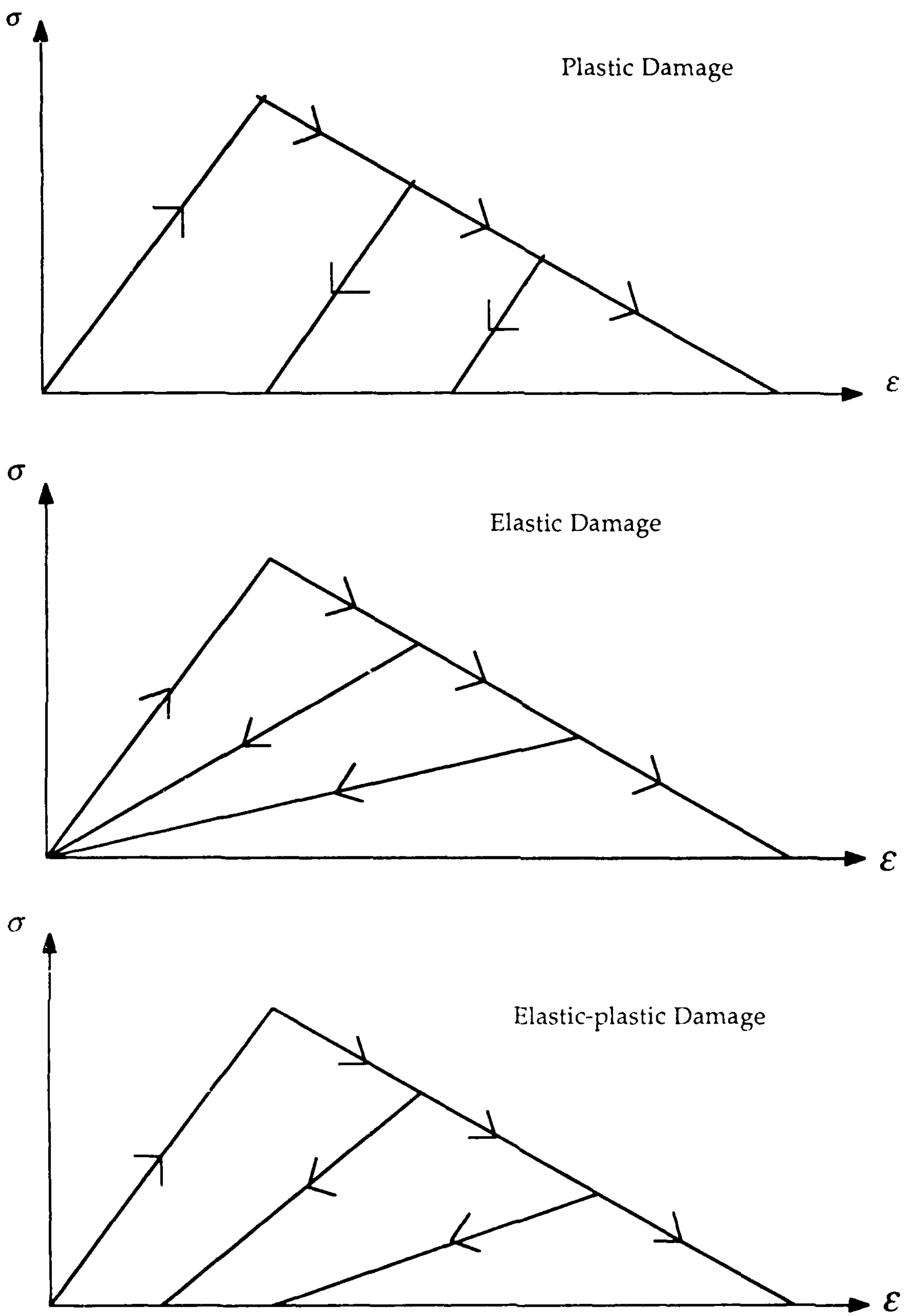
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Figure 3

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